Mathematical Sciences Semesters in Guanajuato

Fall Semester in Mathematical Tools for Modeling
Spring Semester in Mathematical Tools for Data Science
Summer Program in Partial Differential Equations: Theory, Numerical Methods and Applications

Contents

Motivation

Handling, processing, and analyzing voluminous and complex data are now incredibly common tasks that influence many aspects of our everyday life. Thanks to the advent of powerful data collection and storage devices in recent decades, the world is all-connected. Geographical, social, and cultural preferences, and transportation data, to name just a few examples, are being increasingly used to inform decision-making processes, to discover interesting relationships between variables, or to predict future trends. Major industries and government institutions are now forming data science teams to undertake these challenges.

The scientific and technical issues used to address the above-mentioned topic are complex and highly multidisciplinary in nature. In particular, mathematicians are called upon to prescribe, interpret, and supervise quantitative methods of analysis and their corresponding computer implementations.

CIMAT is a federal research and teaching center, founded almost 40 years ago, that has strong experience in the fields of mathematics, statistics, and computer science. It has traditionally fostered an interrelationship culture between its fields of study and it is now offering a program focused on our strengths in mathematics, computer science and statistics. We offer three independent semesters, a Fall semester in Mathematical Tools for Modeling and a Spring semester in Mathematical Tools for Science Data. Additionally, we offer a summer program in Partial Differential Equations: Theory and Applications with Numerical Support.

Each independent semester is aimed at students with a background predominantly in mathematics. The goal is to provide students with a solid and rigorous understanding of the subject's core. The study of data science and modeling with a truly multi-disciplinary and sound approach necessarily involves strong interaction between statistics, mathematical modeling, and computer science. Students will be exposed to concepts and terminology from data science, preparing them to communicate with and take part in multidisciplinary teams. In addition, they will learn the fundamental theoretical bases of quantitative methods, statistical models, and computer science, equipping them with the ability to choose relevant and efficient algorithmic solutions for solving problems in data science and mathematical modeling.

In the last forty years, there has been an accelerated trend of the applicability of mathematics. To the degree that today they flood and permeate our whole life: finance, climate, electronic commerce, communication, new materials are just a few examples.

This success cannot be explained without the equally accelerated computing power now available to us. Combining mathematical techniques and computational power, we can tackle now problems that were absolutely unthinkable only a few years ago. Being able to use mathematical techniques to model real-life situations gives participants in Mathematical Sciences Semesters in Guanajuato (MSSG) a definite advantage in a world in which problems are changing and growing more complex every day.

In each semester, we provide the basic tools to be able to tackle a variety of dynamic problems.

Why come to study Mathematical Tools for Modeling and Data Science in Guanajuato?

- CIMAT is a research center with a 35-year old tradition of **pluri-disciplinary work involving basic mathematics, statistics and computer science**. The presence of these areas makes the perfect setting for the study of Data Science, as the experts in each area will give you the fundamentals to understand data science problems from **both a statistical and an algorithmic point of view.**
- At CIMAT, you will discover how beautiful constructions from **basic mathematics** areas, such as Geometry or Algebra, may help you in understanding data.
- CIMAT has an **extraordinarily vivid academic environment**, with continuous flow of academic events (conferences, seminars and congresses…) in cutting-edge research areas and involving leaders of their fields from the U.S., Canada, Europe and Latin America.
- Guanajuato is one of the best cities to live in Mexico; a place where you can be **immersed in a colorful, joyous, Mexican *ambiente*,** learn or practice Spanish and discover the rich culture of Mexico.

Faculty

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Fall Semester in Mathematical Tools for Modeling

Semester courses:

- Modeling with Differential Geometry
- Discrete Probability and Simulation
- Modeling with Linear Algebra
- Tools for Modeling Dynamics
- Algebra and Its Applications
- Functions of a Complex Variable

Extras:

- Workshop on Computational Tools [2 weeks].

The expected course load for students at MSG is four classes.

Semester Goals/Objectives:

The aim of this semester is to learn and master the mathematical and computational foundations necessary for students majoring in Mathematics, Statistics or Computer Science to deepen their knowledge and practical skills in areas related to modeling. Students are expected to take four courses.

Students should learn the following skills by the end of the semester:

1. Master the basic results in the mathematical areas covered during the semester - linear algebra, differential equations, differential geometry, and probability- needed to develop mathematical models for concrete problems.
2. Master the main software tools related to these fields and be able to solve relevant modeling problems.
3. Be able to take part in multidisciplinary teams to solve modeling problems.
4. Discover aspects of the Mexican culture by immersing themselves in one of its most vibrant historical cities in the heartland of Mexico.

Overall Requisites:

- *Be currently enrolled in a higher education institution, pursuing a major that includes components involving Mathematics, Statistics, Data Science, or Computer Science.*
- *At least one linear algebra course and the standard calculus sequence ending with multivariate calculus.*

- *In calculus, the applicant should be familiar with the notions of integration, derivatives, series, limits;*
- *In linear algebra, the student should be familiar with the concepts of vector spaces, bases, dimensions, matrices, linear transformations, determinants, kernels;*
- *In multivariate calculus, the student should be familiar with the concepts of derivative of a map from R^d to R^k for d and k up to three, chain rule, Hessian, maxima and minima, gradient, cross and dot products.*

Preliminary (optional): Workshop on Computational Tools (2-week workshop)

- Introduction to programming in Python: variables, conditionals, loops, functions, introduction to classes.
- Introduction to programming in R.
- Introduction to SageMath.

Modeling with Differential Geometry

Course Description

Differential geometry uses the tools of multivariate calculus (and linear algebra) to study the "geometry" of non-linear spaces. Roughly speaking, the aim is to study and understand the possible shapes of curves and surfaces in space. Soon enough, a new concept (absent from Euclidean geometry) emerges: curvature; a good part of the course will be devoted to understanding this through examples and classical results.

The first part of the course studies curves, both in the plane and in 3-space, while the second part focuses on the geometry of surfaces in R^3. Throughout the course, we will utilize the computer software package SageMath, both to make long or tedious computations easier and to gain visual intuition.

The course is complemented with examples that show how the mathematics being learned can be used to solve applied problems from different sources. In addition, the last topic of the course will be an introduction to quaternion arithmetic and how to use this formalism to understand rigid transformations of the space and applications to mechanics and robotics.

The goal of this course is for students to achieve a working intuition of the geometry of curves and surfaces (through theorems, examples, the use of visualization tools and applications), that have wide applicability.

Prerequisites

In addition to the common requirements, a first course in ordinary differential equations is recommended.

Course Goals

On completion of the course, students will

- Understand the basic features of curves (in the plane, the space and on surfaces): the invariants that determine them, many examples that are ubiquitous, how to construct and draw them, how to use certain types of curves to interpolate data, how they have been used in applications.
- Have a working intuition of surfaces in 3-space. How to parametrize them, what is the curvature and why it is an obstruction to draw accurate maps. Students will have learned the answers to beautiful classical problems that still serve as guides for contemporary science and technology. They will also have had first-hand experience with examples of surfaces that are used in modeling and applications.

Course Content

1. **Curves** (5 weeks)
   Curves in the plane and space; local theory; global results; applications: involute gears, ribbons and supercoiling of DNA.
2. **Surfaces in 3-space** (7 weeks)
   Parametrized surfaces; first fundamental form (the metric); examples; Gauss map, curvature; ruled and developable surfaces; intrinsic geometry, parallel transport and geodesics; global results, Gauss-Bonnet theorem; applications: hyperboloid gears, an industrial packing problem, rigid body motions, maps (or why is it difficult to paper wrap a ball).
3. **Quaternions** (2 weeks)
   Quaternions, rotations, mechanics and applications to robotics.

Bibliography

1. Shifrin, Theodore (2016). *Differential Geometry: A First Course in Curves and Surfaces.*
2. Oprea, John (2007). *Differential Geometry and its Applications,* Mathematical Association of America.
3. Ghomi, Mohammad. *Lecture Notes on Differential Geometry.*
4. Murray, Richard M.; Zexiang, Li; Shankar Sastry, S. (1994). *A Mathematical Introduction to Robotic Manipulation*, CRC Press.
5. Hilbert, D.; Cohn-Vossen, S. (1999). *Geometry and the Imagination,* AMS Chelsea Publishing.

Support Sessions
2 hours per week with a teaching assistant

Grading
There will be weekly homework counting for 20% of the final grade. The other 80% will be distributed between two midterm exams (20% each) and a final exam (40%)

Discrete Probability and Simulation

Course Description
This course is an introduction to discrete probability focusing on Bernoulli trials and associated distributions. It introduces basic random processes such as the Poisson process, random walk, and Markov chains. The simulation of random variables and random processes is emphasized throughout the course using the software R.

Prerequisites
In addition to the common requirements, students should have taken an elementary probability or probability and statistics course.

Some elementary knowledge of R is also required, but this will be covered during the optional preliminary workshop on computational tools.

Course Goals
On completion of the course, students will be able to

- Understand and use the concepts of probability space, random variable, probability distribution, expected value, independence and random process and will have been exposed to concrete examples of these concepts.
- Handle the main simulation procedures for random variables and use the methods implemented in R to this effect.
- Understand the proof of two basic theorems of probability theory: the law of large numbers and the central limit theorem, in the context of independent Bernoulli trials, and be familiar with some of their applications.
- Define and use random processes such as the Poisson process and discrete Markov chains, as well as use them as models for concrete problems and simulate them to study their properties.
- Understand processes and random variables to model, simulate, and analyze practical problems.

Course Content
1. **Review of basic probability concepts** (3 weeks)

Probability spaces. Conditional probability and independence. Random variables, distribution functions, discrete and continuous variables, expected value, moments, Markov and Chebyshev inequalities. Probability generating functions.

2. **Random variable simulation** (1 1/2 weeks)
   Basic concepts about random number generators. The Inverse Transform method with examples. The acceptance-rejection method with examples. Particular methods: random variables with normal, Poisson, or binomial distribution. Simulation of random variables with R.
3. **Bernoulli trials** (3 weeks)
   Definition and related distributions: Binomial, geometric, negative binomial, Poisson. The Law of Large Numbers, Chernoff bounds, Stirling's formula, the Central Limit Theorem (DeMoivre-Laplace) and applications. Simulation of Bernoulli trials and related variables.
4. **Poisson processes** (2 weeks)
   Exponential distribution. Poisson process, definition and characterizations. Distributions related to a Poisson process. Compound Poisson processes. Decomposition and superposition of Poisson processes. Non-homogeneous processes. Simulation and applications.
5. **Discrete Markov chains** (3 weeks)
   Definition and examples. Transition matrices, Chapman-Kolmogorov equations, classification of states. Stationary distributions. Example: Simple random walk, properties, simulation, Arcsine Law. Simulation of Markov Chains with applications.
6. **Discrete event simulation** (1 1/2 weeks)
   Introduction to the discrete event simulation method. Queue with one server, queue with two serial or two parallel servers, simple inventory models and the insurance risk model. Statistical analysis of simulated data.

Bibliography

1. Baclawski, Kenneth (2008). *Introduction to Probability with R,* Boca Raton: Chapman & Hall/CRC.
2. Dekking, F.M.; Kraaikamp, C.; Lopuhaä, H.P.; Meester, L.E. (2010). *A Modern Introduction to Probability and Statistics*, London: Springer.
3. Dobrow, Robert P. (2014). *Probability with Applications and R,* Hoboken: Wiley.
4. Dobrow, Robert P. (2016). *Introduction to Stochastic Processes with R,* Hoboken: Wiley.
5. Durret, Rick (2009). *Elementary Probability for Applications* Cambridge University Press.
6. Jones, Owen; Maillardet, Robert; Robinson, Andrew (2009). *Introduction to Scientific Programming and Simulation Using R,* Boca Raton: CRC Press.

7. Karlin, Samuel; Taylor, Howard M. (1998) *An Introduction to Stochastic Modeling*, 3rd. Edition, Academic Press.
8. Lesigne, Emmanuel (2005). *Head or Tails: An Introduction to Limit Theorems in Probability,* American Mathematical Society.
9. Ross, Sheldon M. (2012). *Simulation*, 5th. Edition, Academic Press.
10. Suess, Eric A.; Trumbo, Bruce E. (2010). *Introduction to Probability, Simulation and Gibbs Sampling with R,* New York: Springer.

Books on R

1. Adler, Joseph (2010). *R in a Nutshell,* Beijing: O'Reilly.
2. Matloff, Norman (2011). *The Art of R Programming,* San Francisco: No Starch Press.
3. Murrell, Paul (2011). *R Graphics*, 2nd. Edition, Boca Raton: CRC Press.
4. Zuur, Alain; Leno, Elena N.; Meesters, Erik (2009). *A Beginner's Guide to R,* Dordrecht: Springer.

Support Sessions

2 hours per week with a teaching assistant

Grading

Two midterm exams (40%), homework (40%), a simulation project to be submitted/ presented by the end of the course (20%)

Modeling with Linear Algebra

Course Description

This course is about different aspects of linear algebra with emphasis on its applications. Linear algebra unifies the study of linear equations with the geometry of lines and planes. It has great theoretical significance in mathematics and rich applications. The course will start with real-life problems modeled with linear equations, which we will represent using matrices and vectors. Later on, we will deal with topics such as determinants, vector spaces, eigenvalues, and least-square problems. Although the course is mainly focused on modeling, the students will learn the formality and rigor that form a crucial part of algebra. Practical aspects of the course and applications will be emphasized throughout the course by the use of the computer language Python.

Prerequisites

The common requirements for the semester in Mathematical Tools for Modeling.

Course Goals

On completion of the course, students will

- Learn concepts and techniques in linear algebra used for mathematical modeling.
- Identity real-life problems that can be modeled with linear algebra.
- Understand several applications and uses of linear algebra.
- Learn mathematical formality and rigor.

Course Content.

1. **Uses of linear equations and matrices** (4 weeks)
   Review of linear equations and matrix algebra. Linear equations in economics, chemistry, and engineering. Adjacency matrix of a graph. Invertible matrices. Linear algebra and cryptography. Interpolating polynomials. Matrix factorizations. Determinant and its properties. Determinant and volume.
2. **Vector spaces and linear transformations** (4 weeks)
   Vector spaces and subspaces. Spanning sets and linear independence. Basis and dimension of a vector space. Coordinate systems. Linear transformations. Geometry of linear transformations and computer graphics. Kernel and image of a linear transformation. The dimension theorem. Linear isomorphisms. The matrix of a linear transformation. Change of basis and similarity.
3. **Inner product** (3 weeks)
   Inner product and length. Orthogonality. Projections. Applications to machine learning. The Gram-Schmidt process. Least-squares solutions.
4. **Eigenvalues and canonical forms** (3 weeks)
   Review of complex numbers. Eigenvalues and eigenvectors of a matrix. The characteristic equation. Diagonalization of symmetric matrices and the spectral theorem. The Page Rank algorithm. Markov chains.

Bibliography

1. Kwak, Jin Ho; Hong, Sungpyo (2004). *Linear Algebra*, 2nd edition, Springer.
2. Lay, David C.; Lay, Steven R.; McDonald, Judi J. (2016). *Linear Algebra and Its Applications*, 5th edition, Pearson. **[Main textbook]**

Support Sessions

2 hours per week with a teaching assistant

Grading

Midterm exam (25%), final exam (35%), quizzes (15%), homework (10%), a simulation project to be submitted/presented by the end of the course (15%)

Tools for Modeling Dynamics

Course Description

In this course, we will provide mathematical and computational tools for time evolution models of different types of processes.

Throughout the course, there will be a "hands-on" approach utilizing various computer platforms. There will be a two-week workshop on Python before the beginning of the course.

Prerequisites

In addition to the common requirements, a course in ordinary differential equations is required.

Course Goals

By the end of this course, students will

- Understand the basic features of modeling processes that change with time.
- Have a working intuition of different types of dynamics.
- Understand when it is natural to use differential equations, discrete dynamics or partial differential equations.
- Handle the main simulation procedures and use the methods implemented in a computer language.

Course Content

As mentioned above, we will provide mathematical and computational tools for time evolution models of different types of processes.

1. **Differential equations** (4 weeks)
Problems with continuous time are naturally modeled with differential equations. Possible examples: Lorenz attractor, Vander Pol equation, population dynamics, epidemiology.
2. **Difference equations** (3 weeks)
Problems that are modeled in discrete time, equations in differences, logistic model, period bifurcation, Henon attractor. Possible examples: insects' dynamics.
3. **Cellular automata** (3 weeks)
Cellular automata are examples of discrete space and can evolve in time either continuously or also discretely. Possible examples: Infectious processes.
4. **Partial differential equations** (4 weeks)
Possible examples; wave and heat equations.

Bibliography

1. Hirsch, Morris W.; Smale, Stephen; Devaney, Robert L. (2012). *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press
2. Segel, Lee A.; Edelstein-Keshet, Leah (2013). *A Primer on Mathematical Models in Biology,* SIAM.
3. Mickens, Ronald E. (2015). *Difference Equations: Theory, Applications and Advanced Topics*, Third Edition, CRC Press.
4. Kelley, Walter G.; Peterson, Allan C. (2000). *Difference Equations: An Introduction with Applications*, 2nd Edition, Academic Press.
5. Schiff, Joel L. (2008). *Cellular Automata: A Discrete View of the World,* Wiley.
6. Toffoli, Tommaso; Margolus, Norman (1987). *Cellular Automata Machines: A New Environment for Modeling,* MIT Press.
7. Coleman, Matthew P. (2004). *An Introduction to Partial Differential Equations with MATLAB,* 1st Edition, Chapman and Hall/CRC.

Support Sessions
2 hours per week with a teaching assistant

Grading
Two midterm exams (40%), homework (40%), a simulation project to be submitted/presented by the end of the course (20%)

Algebra and Its Applications

Course Description
This course is a first introduction to structures of abstract algebra, specifically, groups, rings, and fields. There will be a significant emphasis on real-life problems which are modeled by abstract algebraic structures.The first quarter of the course will be devoted to elementary number theory. Then, we will focus on group theory. This is the core of this class, and we will devote half of the course to this topic. We will end the class with an overview on rings and fields. Applications will be discussed within each topic.

This class is proof-based. However, it is not expected that you have taken a proof-based course before, and there will be an opportunity to learn and improve proof-writing.

In addition, there will be a significant emphasis on real-life problems which are modeled by abstract algebraic structures.

There will be a final project on applications of group theory (e.g. cryptography, codes, music, physics, and Rubik's cube). Project suggestions will be introduced during the first half of the semester.

Prerequisites

In addition to the common requirements, basic knowledge of sets, functions and equivalence relations is expected.

Course Goals

By the end of this course, students will

- Learn concepts and techniques in group theory.
- Identify real-life problems that can be modeled with abstract algebra.
- Learn mathematical formality and rigor.

Course Content

1. **Basic number theory** (4 weeks)
    a. Postulates for the integers
    b. Mathematical induction
    c. Divisibility
    d. Prime factors and greatest common divisors
    e. Congruence of integers and uses in cryptography and error correcting codes
    f. Arithmetic with congruence classes
2. **Group theory** (5 weeks)
    a. Definition of a group and real-life examples such as Rubik's cube movements
    b. Properties of group elements
    c. Subgroups
    d. Cyclic groups
    e. Homomorphisms
    f. Isomorphisms
    g. Finite permutation groups and uses in physics, music, and entertainment
    h. Cayley's Theorem
    i. Cosets of a subgroup
    j. Normal subgroups
    k. Quotient groups
    l. Direct sums
3. **Rings and fields** (4 weeks)
    a. Definition of a ring
    b. Integral domains and fields

c. The field of quotients of an integral domain
d. Ideals and quotient rings for modeling with polynomials
e. Ring homomorphisms

Bibliography

1. Gilbert, Linda; Gilbert, Jimmie. (2015). *Elements of Modern Algebra*, Eighth Edition, Cengage.
2. Beachy, John A.; Blair, William D. (2005). *Abstract Algebra*, Third Edition, Waveland Press.
3. Adhikari, Mahima R.; Adhikari, Avishek. (2014). *Modern Algebra with Applications*, Springer.

Support Sessions

2 hours per week with a teaching assistant

Grading

Midterm exam (25%), final exam (40%), homework (15%), project (20%)

Functions of a Complex Variable

Course Description

This course is intended to serve as a formal introduction to the theory of functions of a single complex variable, enhancing its analytical and geometrical properties with the use of the computer. Complex variables have influenced a wide range of fields in science and in mathematics: from its applications to engineering, physics and quantum field theory, to helping the development of number theory, dynamical systems and algebraic geometry. This course aims to explain some of the main results for functions of a complex variable while providing sufficient mathematical background to understand computer experiments with conformal maps and iteration of complex polynomials. Visualization of complex functions and their properties will be foster throughout the semester, these computer interactions are directed towards the main application of this course, the exploration of complex dynamical systems.

Prerequisites

In addition to the common requirements, students are expected to be proficient in algebra of complex numbers, sequences and series of numbers and functions, convergence tests, uniform and pointwise convergence. Experience with topology of metric spaces will be helpful, but not expected.

Computer experiments will require some proficiency with Python. Students new to this language are recommended to participate in the workshop on computational tools for modeling.

Course Goals

Successful students will be able to

- Acquire mathematical formality and develop their skills in proof writing.
- State the main differences between a differentiable function in two real variables and a holomorphic function and state the connection among power series expansions, a (complex) analytic function and a meromorphic function.
- Develop the theory of contour integration, state and prove Cauchy's Theorem and apply it to compute integrals of real and complex variables.
- Gain geometrical intuition behind conformal maps and normal families.

Course Content

1. **Complex plane and elementary functions** (1 week)

Complex numbers, representations of complex numbers, stereographic projection, exponential and logarithm, powers and roots, phase visualization.

2. **Complex derivative** (3 weeks)

Derivative of a complex function, Cauchy-Riemann equations and holomorphic functions, inverse mapping theorem and the Jacobian, harmonic functions and conjugates, conformal maps and fractional linear transformations. Computer experiments.

3. **Complex integration** (4 weeks)

Multi-calculus review, line integrals and Green's Theorem. Fundamental Theorem of Calculus for holomorphic functions, Cauchy Theorem, Cauchy Integration Formula, Liouville's Theorem, Morera's Theorem and maximum modulus principle.

4. **Power series and residues** (4 weeks)
   Sequences and series of functions. Power series and analytic functions. Power series as holomorphic functions. Meromorphic functions, Laurent series and isolated singularities. Residue Theorem and computation of integrals using residues.
5. **Normal families and iteration of polynomials** (2 weeks)

Uniform and normal convergence of sequences. Fatou and Julia sets for iterated polynomials. The Mandelbrot set and computer experiments.

Bibliography

1. Gamelin, Theodore W. (2001). *Complex analysis*, Undergraduate texts in mathematics, Springer.
2. Wergert, Elias (2012). *Visual complex functions*, Birkhauser.

3. Freitag, Eberhard; Busam, Rolf (2009). *Complex analysis*, Universitext, Springer-Verlag.
4. Marsden, Jerrold E.; Hoffman, Michael J. (1998). *Basic complex analysis*, W. H. Freeman & Co.

Support Sessions
Two hours per week with a teaching assistant

Grading
Two midterm exams (30% each) and weekly homework assignments from which, the best ten assignments will be considered to obtain the remaining 40% of your final grade

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Spring Semester in Mathematical Tools for Data Science

Semester courses:

- Design of Algorithms
- Probability and Statistics
- Numerical Optimization and Machine Learning
- Discrete Mathematics
- Geometric and Graphing Tools

Extras:

- Workshop on Computational Tools [2 weeks].

The expected course load for students at MSG is four classes.

Semester Goals/Objectives:

The aim of this semester is to learn and master the mathematical and computational foundations necessary for students to deepen their knowledge and practical skills in areas related to data science.
Students should master the following skills by the end of the semester:

1. Master the mathematical foundations of data science; i.e., the most important concepts in discrete mathematics, rigorous proof techniques and multivariate

statistics in order to provide sound mathematical models for data science problems.
2. Master the algorithmic foundations of data science (e.g., the basic tools to quantify algorithm complexity) and be able to identify canonical algorithmic problems and propose adequate algorithm paradigms to solve them.
3. Be able to effectively participate in a multi-disciplinary team involving statisticians, mathematicians, computer scientists, and specialists from other areas to solve a data science problem as a team.
4. Discover different aspects of the Mexican culture, by immersing themselves in one of its most vibrant historical cities in the heartland of Mexico.

Overall Requisites:

- *Be currently enrolled in a higher education institution, pursuing a major that includes components involving Mathematics, Statistics, Data Science, or Computer Science.*
- *At least one introductory programming course. The applicant should understand the notions of control structures, conditionals, variables, and functions.*
- *Al least one linear algebra course. The student should be familiar with the concepts of vector spaces, bases, dimensions, matrices, linear applications, determinants, and kernels.*
- *Differential, integral and multivariate calculus courses. The applicant should be familiar with the notions of limits, integration, derivatives, and series.*

Preliminary (optional): Workshop on Computational Tools (2-week workshop)

- Introduction to programming in Python: variables, conditionals, loops, functions, introduction to classes.
- Introduction to programming in R.
- Introduction to SageMath

Design of Algorithms

Course Description

In this course, we will review the fundamentals in the design and rigorous analysis of algorithms oriented toward the solution of data science problems. The first part of the course will be dedicated to reviewing elementary data structures and algorithm paradigms; we will then focus on algorithms on graphs, matching problems, searching problems, applications to data mining and on computational geometry.

Prerequisites
Common requirements for the Semester in Mathematical Tools for Data Science and some knowledge of discrete mathematics.

Course Goals
On completion of the course, students will be able to

- Master fundamental concepts necessary to analyze algorithms in terms of complexity.
- Be able to read and reproduce proofs of correctness of algorithms and understand the different methodologies to implement them.
- Master the essential data structures and main algorithms for dealing with graphs, recursive problems and computational geometry.
- Discover the algorithmic nature of several problems involved in data science.
- Learn to implement rigorously and methodically an algorithm in a high-level programming language.

Course Content

1. **General tools for studying algorithms** (3 weeks)
    a. Algorithm/problem complexity. Evaluation of algorithm complexity.
    b. Elementary data structures (stacks, queues, binary trees, heaps, priority queues, disjoint sets).
2. **Algorithms on data sequences** (3 weeks)
    a. Divide and conquer and dynamic programming, and applications to data science.
    b. Sequence alignment problems.
3. **Algorithms on graphs** (5 weeks)
    a. Graphs in data science. Data structures for graphs, traversal algorithms and applications.
    b. Algorithms on graphs: shortest paths; Bellman-Ford; Dijkstra; A*.
    c. Algorithms on graphs: flow networks; applications to bipartite graph matching.
    d. Applications to Data Science.
4. **Handling large datasets** (3 weeks)
    a. Data mapping and dictionaries; hashing.
    b. Similarity search.
    c. Randomized and memory-free methods.

Bibliography

1. Cormen, Thomas H.; Leiserson, Charles E.; Rivest, Ronald L.; Stein, Clifford, (2009). *Introduction to Algorithms,* 3rd Edition, MIT Press. [Main textbook]
2. Cormen, Thomas H. (2013). *Algorithms Unlocked*, MIT Press.
3. Kleinberg, Jon; Tardos, Eva (2005). *Algorithm Design*, Addison-Wesley.
4. Leskovec, Jure; Rajaraman, Anand; Ullman, Jeff (2014). *Mining Massive Datasets*, Cambridge University Press.
5. Hromkovic, Jurj (2005). *Design and Analysis of Randomized Algorithms*, Springer.

Support Sessions

2 hours a week with a teaching assistant

Grading

Two midterm exams (10% each), homework assignments (40%), final exam (20%), integrative project (20%)

Numerical Optimization and Machine Learning

Course Description

This course aims at providing students with relevant modern computational numerical techniques using linear algebra, optimization and machine learning, so the undergraduate will be able to attack data science and machine learning problems in an integrated manner.

Prerequisites

Common requirements for the semester and some knowledge of discrete mathematics.

Course Goals

On completion of the course, students will

- Master the most important concepts from numerical methods and numerical optimization to understand the deep roots of machine learning in these areas.
- Be proficient with numerical tools to develop intuition in data science and statistics, as compared to analytical methods.
- Understand the importance of the implementation of numerical solutions for data exploration.
- Be able to take a multidisciplinary approach to data science projects, involving methodologies from computational statistics and numerical analysis.

Course Content

1. **Numerical optimization and data science** (7 weeks)
    a. Linear systems: Iterative methods (e.g., Gauss-Seidel), matrix spectrum (eigenvalues and eigenvectors), matrix factorization (e.g., SVD), overdetermined systems (least squares). Interpolation and curve fitting (splines, optional).
    b. Generalities about numerical optimization and machine learning. Gradient descent. Stochastic gradient descent. Conjugate gradient. Metaheuristics: Search spaces, neighborhoods, sampling of the search space: exploration and exploitation, meta-modeling.
2. **Introduction to machine learning** (7 weeks)
    a. Overview of models and challenges in machine learning and data science: geometric vs. probabilistic approaches; predictive models vs. inference models.
    b. Probabilistic methods: Bayesian classification (naive/optimal).
    c. Geometrical methods: Knn, decision trees.
    d. Margin-based methods. SVMs, kernel methods.
    e. Neural networks.
    f. Introduction to ensemble methods.

Bibliography

1. Eldén, Lars (2007). *Matrix Methods in Data Mining and Pattern Recognition (Fundamentals of Algorithms),* SIAM.
2. Nocedal, Jorge; Wright, Stephen J. (2006). *Numerical Optimization*, 2nd Edition, Springer.
3. Bishop, Christopher M. (2006). *Pattern Recognition and Machine Learning,* Springer.
4. Bengio, Yoshua; Goodfellow, Ian; Courville, Aaron (2016). *Deep Learning*, MIT Press.
5. Gareth, James; Witten, Daniela; Hastie, Trevor; Tibshirani, Robert (2013). *Introduction to Statistical Learning*, Springer.

Support Sessions

2 hours a week with a teaching assistant

Grading

Partial exams in each block (25%), projects in each block (25%), homework assignments (50%)

Probability and Statistics

Course Description

This course is an introduction to statistical thinking and concepts, beginning with basic probability theory. The course concludes with selected statistical methods useful for data exploration and description of vector-valued data, a common setup in modern data analysis applications. Python and/or R will be used for practical implementation of all numerical and graphical procedures, including simulations.

Prerequisites

Common requirements for the Semester in Mathematical Tools for Data Science.

Course Goals

On completion of the course, students will

- Learn about basic statistical concepts and methods, including uncertainty and the role of probabilistic reasoning in data analysis.
- Master presentation and use of mathematical concepts in probability theory.
- Learn about selected methods for addressing statistical problems, such as multiple linear regression and logistic regression for issues in inferring about data structure, prediction, and classification.
- Implement methods and graphical procedures via Python, using meaningful datasets.

Course Content

1. **Introduction** (0.5 week)
   Statistical thinking, role of data, stochasticity, and uncertainty.
2. **Probability theory** (2 weeks)
   Sample space and events. Basic properties of probability. Probability laws. Conditional probability and independence. Bayes Theorem.
3. **Random variables** (2.5 weeks)
   Mean and variance. Discrete families: Bernoulli, binomial, geometric and Poisson densities. Continuous families: exponential and normal densities. Multivariate normal distribution.
4. **Graphical methods for exploring univariate and multivariate data** (1.5 weeks)
   Graphical tools for multivariate descriptions (matrix plots, parallel plots, icon plots, etc.).
5. **Statistical inference** (4.5 weeks)
   Likelihood. Asymptotic normality of maximum likelihood estimators. Bootstrap.

Bayesian inference. Elements of Bayesian inference via MCMC (Markov Chain Monte Carlo).

6. **Regression models** (3 weeks)
   Linear regression and logistic regression. Prediction and classification.

Bibliography

1. Baron, Michael. (2014). *Probability and Statistics for Computer Scientists*, 2nd Edition, CRC Press.
2. DeGroot, Morris H.; Schervish, Mark J. (2012) *Probability and Statistics*, Addison-Wesley. **[Main textbook]**
3. Wasserman, Larry. (2004). *All of Statistics: A Concise Course on Statistical Inference*, Springer.
4. Cook, Dianne; Swayne, Deborah F. (2007) *Interactive and Dynamical Graphics for Data Analysis: With R and GGobi*, Springer.

Support Sessions

2 hours a week with a teaching assistant

Grading

Two midterm exams (25% each), homework (20%) and a final project (30%)

Discrete Mathematics

Course description

This course introduces the student to the basic subjects of discrete mathematics, which forms the mathematical foundation of computer and information science. The course will include small projects in SageMath.

Prerequisites

Common requirements and some knowledge of elementary combinatorics and finite sample spaces

Course Goals

On completion of the course, students will

- Strengthen mathematical thinking and problem-solving abilities.
- Know a range of discrete models and topics which are found in applications in modern fields like data science.
- Develop confidence in working with discrete mathematical objects on a computer programming language.

Course Content

This course is intended for 14 weeks which include 5 small computer labs.

1. **Logic, proofs, and induction** (2 weeks)
   Elements of logic. Basic proof techniques. Proof by cases and geometry. Induction, Well-order. Inductive definitions.
2. **Sets, functions and relations** (1 week)
   Relations (ordered pairs). Functions. Data bases and sets.
   Computer lab: Programming functions.
3. **Recurrence and generating functions in algorithms** (2 weeks) Recurrences and recursive functions. Fibonacci numbers. Linear recurrence relations with constant coefficients. Rational power series. Arrangements and involutions. Generating functions. Exponential generating function.
   Computer lab: Multiplying polynomials and formal series.
4. **Number theory** (2 weeks)
   Divisibility. Modular arithmetic. Primes. Euler function. RSA cryptosystem.
   Computer lab: Encryption.
5. **Counting** (2 weeks)
   Factorial and binomial coefficients. Pascal triangle. Permutations and combinations. Combinatorial identities. Inclusion-exclusion principle. Double counting and pigeonhole principle.
6. **Combinatorics and probability** (1 week)
   Computing probabilities in classical problems. Bernoulli trials. Law of large numbers. Central limit theorem.
   Computer lab: Playing with Pascal triangle.
7. **Trees** (1 week)
   Expanding trees. Binary trees. Rooted trees.
8. **Advanced graph theory** (2 weeks)
   Graph coloring. Connectivity. Hamiltonian and Eulerian paths. Graph isomorphism. Introduction to spectral graph theory.
   Computer lab: Generating random graphs.
9. **Probabilistic methods** (1 week)
10. **Basic inequalities in probability**. First moment method. Second moment method.

Bibliography

1. Alon, Noga; Spencer, Joel H. (2004). *The Probabilistic Method*, John Wiley & Sons.
2. Newman, Mark (2010). *Networks: an Introduction*, Oxford University Press.

3. Rosen, Kenneth H. (2007). *Discrete Mathematics and its Applications*, McGraw-Hill Education.
4. Wilf, Herbert S. (2006). *Generating Functionology*, CRC Press.
5. Anderson, Ian (1989). *Combinatorics of Finite Sets*, Oxford University Press.
6. Diestel, Reinhard (1994). *Graph Theory*, Graduate Texts in Mathematics, Springer
7. Knuth, Donald E.; Graham, Ronald L.; Patashnik, Oren (1994). *Concrete Mathematics: A Foundation for Computer Science*, 2nd. Edition, Pearson.
8. Lovász, László; Pelikán, József; Vesztergombi, Katalin (1999). *Discrete Mathematics: Elementary and Beyond*, Springer

Support Sessions
2 hours per week with a teaching assistant

Grading
Two midterm exams (40%), homework (40%), a simulation project to be submitted/presented by the end of the course (20%)

Geometric and Graphing Tools

Course Description
This course is an introduction to the fundamentals of data visualization, interactive data exploration and storytelling. The second part of this course includes basic algorithms and data structures from computational geometry useful for data visualization.

The course will make an overview of available software and programming languages for data visualization.

Prerequisites
Common requirements for the semester and some knowledge of algorithmic complexity.

Course Goals
On completion of the course, students will

- Master the fundamentals of visualization techniques for data science.
- Be acquainted with the basic geometric data structures that are useful in data visualization.
- Master basic geometrical algorithms and understand their complexities.
- Be familiar with the usage of current visualization software for basic geometrical display.

Course Content
This course is intended for 14 weeks.

1. **Fundamentals of data visualization** (5 weeks)
   a. Principles of data visualization, examples of good and bad data visualizations.
   b. Introduction to human perception of 2d and 3d images.
   c. Fundamentals of 2d and 3d computer graphics for visualization: computer graphics pipeline, 2d and 3d image generation.
   d. Types of data and their visualizations.
2. **Computational Geometry for data visualization** (6 weeks)
   a. Introduction to algorithms with geometrical structures: examples, complexity, basic constant operations.
   b. Voronoi diagrams.
   c. Polygon triangulation, delaunay triangulation.
   d. Orthogonal range search.
3. **Working with data** (3 weeks)
   a. Visualization examples.

Bibliography

1. Tufte, Edward R. (2001). *The Visual Display of Quantitative Information*, 2nd Edition, Graphics Press.
2. Tufte, Edward R. (1990). *Envisioning Information*,. Graphics Press.
3. Munzner, Tamara. (2014). *Visualization Analysis & Design*,. CRC Press.
4. Ware, Colin (2012). *Information Visualization: Perception for Design*,. Morgan Kaufmann.
5. De Berg, Mark; Cheong, Otfried; Van Kreveld, Marc; Overmars, Mark (2008). *Computational Geometry, Algorithms and Applications*, Springer.
6. Boissonnat, Jean-Daniel; Chazal, Frédéric; Yvinec, Mariette (2016). *Computational Geometry and Topology for Data Analysis.*
7. Devadoss, Satyan L.; O'Rourke, Joseph (2011). *Discrete and Computational Geometry,* Princeton University Press.

Support Sessions
2 hours a week with a teaching assistant.

Grading
Midterm exam (20%), homework assignments (40%), final exam (20%), integrative project (20%).

Summer Program in Partial Differential Equations: Theory, Numerical Methods and Applications

Courses:

- Differential Equations in Numerical Modeling
- Linear Partial Differential Equations
- Mathematical Finance

Extras:

- Workshop on computational tools

Program Goals/Objectives

The purpose of this summer program is to build a solid foundation on the theory and applications of Partial Differential Equations (PDE). The aim is to highlight the multidisciplinary features of PDE. Consequently, applications in diverse fields are covered, in particular Mathematical Finance. A thorough exposition of the subject is presented and PDE models are explored through numerical simulation. The goal is not only to introduce the numerical methods of solution, but to explore the underlying analysis.

Overall Requisites:

- *Be currently enrolled in a higher education institution, pursuing a major that includes components involving Mathematics, Statistics, Data Science, or Computer Science.*
- *At least one linear algebra course and the standard calculus sequence ending with multivariate calculus.*
- *An elementary probability or probability and statistics course.*
- *An ordinary differential equations course.*

Faculty

Chang Lara, Héctor Andrés -CIMAT, Mathematics
Hernández Hernández, Daniel -CIMAT, Probability and Statistics
Moreles Vázquez, Miguel Ángel -CIMAT, Mathematics

Preliminary (optional): Workshop on Computational Tools (1-week workshop)

- Introduction to programming in MATLAB.

Differential Equations in Numerical Modeling

Course Description

The purpose of the course is to present numerical methods for solving differential equations arising from modeling real-world problems. The underlying mathematics of the numerical methods are an integral part of the course. Each chapter starts with the derivation of a model for a physical problem in terms of differential equations. Then a numerical method for solution is introduced and analyzed.

Prerequisites

Common requirements for the summer program

Course Goals

On completion of the course, students will

- Understand the basics of mathematical modeling. From physical problem to equation.
- Know the properties of numerical methods to make informed choices for solution.
- Be able to do numerical simulation in a software environment such as Matlab.
- Be proficient in the underlying mathematics of numerical methods.

Course Content

1. Ordinary differential equations. Embedded Runge-Kutta methods. Parameter estimation in ODE (1 week).
2. Finite difference method. Potential and diffusion (1 week).
3. Finite volume method. Examples in Computational Fluid Dynamics (CFD) (2 weeks).
4. The finite element method for elasticity (2 weeks).
5. Discontinuous Galerkin method. Scalar conservation laws (2 weeks).

Bibliography

1. Cooper, Jeffery M. (2000). *Introduction to Partial Differential Equations with MatLab*, Boston: Birkhäuser.

2. Van Groesen, E.; Molenar, Jaap (2007). *Continuum Modeling in the Physical Sciences*, Philadelphia: SIAM.
3. LeVeque, Randall J. (2007). *Finite Difference Methods for Ordinary and Partial Differential Equations, Steady-State and Time-Dependent Problems*, Philadelphia: SIAM.
4. Mattheij, R.M.M.; Rienstra, S.W.; ten Thije Boonkkamp, J.H.M. (2005). *Partial Differential Equations; Modeling, Analysis, Computation,* Philadelphia: SIAM.
5. Quarteroni, Alfio; Sacco, Riccardo; Saleri, Fausto (2000). *Numerical Mathematics*, New York: Springer.
6. Stoer, Josef; Bulirsch, R. (2002). *Introduction to Numerical Analysis*, Third Edition, New York: Springer-Verlag.

Support Sessions
2 hours per week with a teaching assistant

Grading
Homework (40%), midterm exam (30%), final project (30%)

Linear Partial Differential Equations

Course Description
This course covers the main families of linear partial differential equations: transport, wave, elliptic, and parabolic. For each one of them we present methods to compute exact solutions based on the linear structure of the problem. We will also analyze qualitative properties of the solutions, fundamental to understanding the challenges encountered in their corresponding numerical models.

Prerequisites
Common requirements for the summer program

Course Goals
On completion of this course, students will

- Understand some of the basic techniques for analyzing the major families of partial differential equations: elliptic, parabolic, and hyperbolic.
- Be fluent in linear methods such as the Fourier transform and the fundamental solution.
- Have an overview of some the major applications in engineering, physics, and finance.

Course Content

1. **Introduction** (1 week)
   First order linear equation and the method of characteristics. Geometric interpretation of the gradient, divergence and the Laplacian.
2. **The Fourier transform** (2 weeks)
   One dimensional heat equation. One dimensional wave equation. Separation of variables.
3. **Wave equation** (1 week)
   d'Alembert's formula. Non-homogeneous problem. Maxwell equations.
4. **Laplace equation** (2 weeks)
   Fundamental solution. Mean value formula. Green's function and Poisson's kernel. Maximum principle. Dirichlet principle. Relation with Brownian motion.
5. **Heat equation** (2 week)
   Fundamental solution. Mean value formula. Green's function and Poisson's kernel. Maximum principle. Dirichlet principle. Black-Scholes equation.

Bibliography

1. Myint-U, Tyn; Debnath, Lokenath (2007). *Linear Partial Differential Equations for Scientists and Engineers* (eBook), Boston: Birkhäuser.
2. Evans, Lawrence C. (2010). *Partial differential equations*, Second Edition, Graduate Studies in Mathematics, 19. Providence, RI: American Mathematical Society.
3. Cooper, Jeffery M. (2000): *Introduction to Partial Differential Equations with MatLab*, Boston: Birkhäuser.
4. Haberman, Richard (2013). *Applied partial differential equations with Fourier series and boundary value problems*, Fifth Edition, Upper Saddle River, NJ: Pearson Education, Inc.

Support Sessions

2 hours per week with a teaching assistant

Grading

Homework (30%), midterm (30%), final exam (40%)

Mathematical Finance

Course Description.

The aim of this course is to provide the mathematical foundations of asset pricing valuation in different frameworks. The characteristics of a large class of contracts will be analyzed, and the methodology to provide an arbitrage-free value will be presented. In addition, the basic binomial and Black-Scholes models will be used during the course to analyze the solution of the optimal investment and consumption problems, motivating the connection with solutions of partial differential equations.

Prerequisites
Common requirements for the summer program

Course Goals
On completion of the course, students will

- Have a broad view of the type of derivatives and structured contracts delivered in the financial markets.
- Define a methodology to value this class of derivatives for complete financial markets in discrete time, implementing concepts like hedging price and absence of arbitrage opportunity.
- Propose different models to describe the evolution of stock prices, in discrete time, based on Markov chains.
- Describe the hedging strategy for derivatives in complete markets, in particular in the binomial model.
- Describe the evolution of the fundamental assets in the risk-neutral framework.
- Develop numerical techniques to calculate the arbitrage-free price of derivatives using Monte Carlo methods.

Course Content

1. **Fundamental elements of a financial market**. (1 week)
   Introduction to financial markets. Different contracts based on risky assets. Structured products on indexes, definitions and characteristics, rules of the markets. Interest rates.
2. **Risk-neutral valuation and arbitrage considerations**. (1 week)
   Arbitrage and risk-neutral probability measures. Valuation of contingent claims. Single-period models. Valuation and hedging in complete and incomplete markets. Investment strategies.
3. **Multiperiod financial markets**. (2 weeks)
   Conditional expectation and martingales in discrete time. Optimal portfolios for the binomial model. Markov models and incomplete markets. Valuation of European options. Cox Ross and Rubinstein model.
4. **American options** (2 weeks)
   Stopping time. Snell envelope and decomposition of supermartingales.
   Valuation of American options. Perpetual and finite horizon cases. Hedging portfolios in complete markets.

5. **Dynamic programming** (2 week)
   Optimal portfolios and dynamic programming. Martingale methods for solving optimal consumption problems. Optimal portfolios with constraints.

Bibliography

1. Baxter, Martin; Rennie, Andrew (1996). *Financial Calculus, An Introduction to derivative pricing,* Cambridge University Press.
2. Duffie, Darrell (2001). *Dynamic Asset Pricing Theory, 3rd Edition*, Princeton University Press.
3. Lamberton, Damien; Lapeyre Bernard (2007). *Introduction to Stochastic Calculus Applied to Finance,* 2nd Edition, Chapman & Hall/CRC.
4. Pliska, Stanley R. (1997). *Introduction to Mathematical Finance*, Wiley.
5. Lapeyre, Bernard; Sulem, Agnes; Taley, Denis. *Simulation of Financial Models: Mathematical Foundations and Applications*, Cambridge University Press.
6. Shreve, Steve (2004). *Stochastic Calculus for Finance. Volume I – The Binomial Asset Pricing Model*, Springer.
7. Shreve, Steve (2010). *Stochastic Calculus for Finance. Volume II – Continuous Time Models*, New York: Springer- Verlag.
8. Wilmott, Peter (1998). Derivatives. *The Theory and Practice of Financial Engineering*, Wiley.

Support Sessions

2 hours per week with a teaching assistant

Grading

Homework (30%), midterm exam (30%), final exam (40%)

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