Stochastic simulation using probability models

Intensive Short-Courses

Instructor:
TBA

Hours:
45 hours

Description:

Computer simulation is a powerful and flexible tool for modeling complex systems aimed at understanding their behavior and studying predictions under arbitrary "what if" scenarios. The course focuses on simulation in settings that may involve random inputs and outputs. Examples abound in many applications of mathematics (biology, economics, industry, social sciences and many others). Mathematically, the course includes a sound introduction to discrete probability models focusing on Bernoulli trials and associated distributions. Basic random processes such as the Poisson process, the random walk, and Markov chains are discussed. Modeling aspects are also emphasized, in the sense of establishing why certain probability models are suitable in given situations and how they are elicited based on data analysis. The simulation of random variables and random processes is implemented throughout the course using the R programming language.

Overall requirements:

- Your intended major should include components involving Mathematics, Statistics, Data Science, or Computer Science.
- Students should have taken an elementary probability or probability and statistics course.
- Elementary experience in coding and some knowledge of R are also required, at the level of being able to write scripts for specific tasks involving data input and output, basic computations and plotting.

Stochastic simulation using probability models

Intensive Short-Courses

Instructor:
TBA

Hours:
45 hours

Course Goals:

On completion of the course, students will

- To understand the mathematical concepts of random variables and their characterization via density functions, and to become fluent in basic derived concepts such as expected value, moments, independence and random processes.
- To understand some "behind-the-scenes" procedures that computers employ for the simulation of random variables, and to use functions available in R to implement random number generation.
- To understand the proof and empirically verify a basic theorem in probability theory that is especially relevant for computer simulation: the Law of large numbers.
- To define and perform data-based elicitation of simple probabilistic models as well as more complex ones such as Poisson processes and discrete Markov chains, engaging them as components in complex simulation models.
- To study complex systems by means of stochastic simulation models.

Stochastic simulation using probability models

Intensive Short-Courses

Instructor:
TBA

Hours:
45 hours

Course Content:

1. Review of basic probability concepts

Probability spaces. Conditional probability and independence. Random variables, distribution functions, discrete and continuous variables, expected value, moments, Markov and Chebyshev inequalities. Law of large numbers.

2. Random variable simulation

Basic concepts about random number generators. The Inverse Transform method with examples. The acceptance-rejection method with examples. Particular methods: random variables with normal, Poisson, or binomial distribution. Simulation of random variables with R.

3. Bernoulli trials

Definition and related distributions: Binomial, geometric, negative binomial, Poisson. The Law of Large Numbers, the Central Limit Theorem and applications. Simulation of Bernoulli trials and related variables.

4. Poisson processes

Exponential distribution. Poisson process, definition and characterizations. Distributions related to a Poisson process. Compound Poisson processes. Decomposition and superposition of Poisson processes. Non-homogeneous processes. Simulation and applications.

5. Discrete Markov chains

Definition and examples. Transition matrices. Stationary distributions. Example: Simple random walk, properties, simulation. Simulation of Markov Chains with applications.

6. Discrete event simulation

Introduction to the discrete event simulation method. Queue with one server, queue with two serial or two parallel servers, simple inventory models and the insurance risk model. Statistical analysis of simulated data.

Stochastic simulation using probability models

Intensive Short-Courses

Instructor: TBA

Hours: 45 hours

Grading:

- Homework (40%)
- A simulation project to be submitted/ presented by the end of the course (60%)

Bibliography:

1. Baclawski, Kenneth (2008). Introduction to Probability with R, Boca Raton: Chapman & Hall/CRC.
2. Dekking, F.M.; Kraaikamp, C.; Lopuhaä, H.P.; Meester, L.E. (2010). A Modern Introduction to Probability and Statistics, London: Springer.
3. Dobrow, Robert P. (2014). Probability with Applications and R, Hoboken: Wiley.
4. Dobrow, Robert P. (2016). Introduction to Stochastic Processes with R, Hoboken: Wiley.
5. Durret, Rick (2009). Elementary Probability for Applications, Cambridge University Press.
6. Jones, Owen; Maillardet, Robert; Robinson, Andrew (2009). Introduction to Scientific Programming and Simulation Using R, Boca Raton: CRC Press.
7. Karlin, Samuel; Taylor, Howard M. (1998) An Introduction to Stochastic Modeling, 3rd. Edition, Academic Press.
8. Lesigne, Emmanuel (2005). Head or Tails: An Introduction to Limit Theorems in Probability. American Mathematical Society.
9. Ross, Sheldon M. (2012). Simulation, 5th. Edition, Academic Press.
10. Suess, Eric A.; Trumbo, Bruce E. (2010). Introduction to Probability, Simulation and Gibbs Sampling with R, New York: Springer.

Stochastic simulation using probability models

Intensive Short-Courses

Instructor:
TBA

Hours:
45 hours

Books on R

1. Adler, Joseph (2010). *R in a Nutshell,* Beijing: O'Reilly.
2. Matloff, Norman (2011). *The Art of R Programming,* San Francisco: No Starch Press.
3. Murrell, Paul (2011). *R Graphics,* 2nd. Edition, Boca Raton: CRC Press.
4. Zuur, Alain; Leno, Elena N.; Meesters, Erik (2009). *A Beginner's Guide to R,* Dordrecht: Springer.